PHYSICAL REVIEW E **79**, 026611 (2009)

Nonlinear Fano-Feshbach resonances

Andrey E. Miroshnichenko*
Nonlinear Physics Centre and Centre for Ultra-high bandwidth Devices for Optical Systems (CUDOS), Australian National University, Canberra ACT 0200, Australia
(Received 15 September 2008; published 27 February 2009)

We study the wave scattering in an one-dimensional discrete system with two side-coupled defects. Each of the defects exhibits the Fano resonance as a resonant suppression of transmission, i.e., resonant reflection. We demonstrate that the interaction between two Fano resonances may give rise to a birth of a very narrow resonance. This effect may be understood in terms of the overlapping resonances, as suggested by Feshbach [Ann. Phys. **5**, 357 (1958)]. We consider two cases, when the defects are coupled either locally or nonlocally to the discrete array. In the latter case, a sharp asymmetric resonance appears with a large quality factor. We demonstrate that by introducing a nonlinearity at side-coupled defects a closed loop in the nonlinear transmission coefficient may appear, which results in bistable response.

DOI: 10.1103/PhysRevE.79.026611 PACS number(s): 42.25.Bs, 42.65.Pc, 42.65.Wi

I. INTRODUCTION

The Fano resonance is widely known across various branches of physics. Its main peculiarity is the resonant suppression of the transmission, which was for the first time observed by Wood in the spectrum of light resolved by optical diffraction grating back in 1902 [1], but was not explained properly at that time and is known as Wood's anomaly [2]. Only 60 years later Fano gave an explanation of similar phenomena in quantum physics in terms of constructive and destructive interferences [3]. When these interferences take place close to each other, the formation of a sharp asymmetric response occurs. Fano has also suggested the simplest model, which supports such kind of asymmetric response, namely, he has considered a situation when a degenerated discrete state is coupled to a continuum [3]. This model can be described by a one-dimensional system with a side-coupled defect [4]. One of the advantages of this model is that it can be solved analytically, and, therefore, can help to shed a light on the origin of the phenomenon. Indeed, such a simple model has been successfully applied to explain the appearance of the Fano resonance in rather complex systems. In particular, the light absorption by atomic systems [3], the Aharonov-Bohm interferometer [5,6], quantum dots [7–9], light propagation in a variety of photonic circuits [10–17], phonon scattering by time-periodic scattering potentials [18–20], scattering of atoms in Bose-Einstein condensate in optical lattices [21–23], and many others.

Recently, it was demonstrated that in some nonlinear optical systems [24–28] light propagation exhibits the so-called nonlinear Fano resonances [4], where the transmission vanishes for certain values of the input power. Moreover, the nonlinearity may provide with the bistable response of the system. The main advantage of the nonlinear Fano resonance is a possibility to achieve the bistable transmission at very low input powers, due to a large quality factor of the resonance, which has been successfully demonstrated in terms of all-optical switching operation in photonic crystal circuits [14–17].

The aim of this paper is to study the interaction of two Fano resonances, based on this simple model with two side-coupled defects. It is known, that at the Fano resonance there is a π-jump of the phase of the scattering wave, leading to complete destructive interference, and, therefore, to total suppression of the transmission [4]. When two Fano resonances are located very close to each other, two π-jumps should happen one after another. We demonstrate, that in this case the phase cancelation may occur resulting in constructive interference condition, giving rise to a birth of total resonant transmission. The similar phenomenon of overlapping resonances was studied by Feshbach [29,30], so we call such kind of resonances as Fano-Feshbach resonances. We consider two different cases, when two defects are coupled either locally (to one site), or nonlocally (to two sites) in the discrete array, and study the role of nonlocality. Our results suggest that the nonlocal case is much more attractive for possible applications, since it may provide with much narrower resonances than the local case for the same parameters. As a next step, we consider the nonlinear response of the system, by introducing a Kerr type of nonlinearity to one of the side-coupled defects. We demonstrate that in this case the dependence of the nonlinear transmission coefficient on input power exhibits the nonlinear Fano-Feshbach resonance, when the transmission changes from zero to one, and vice versa. One of the peculiarities of this kind of nonlinear transmission is the presence of a closed loop responsible for bistability.

The paper is organized as follows. In Sec. II the local coupling case is considered. The nonlocal coupling case is studied in Sec. III, first in the linear regime (Sec. III A) and then in the nonlinear regime (Sec. III B). Section IV concludes the paper.

II. LOCAL COUPLING

One of the simplest models, which exhibits the Fano resonances is a one-dimensional discrete chain with one side-coupled defect [4]. In order to study the interaction of two Fano resonances, we generalize this model by introducing the second defect [31].

*aem124@rsphysse.anu.edu.au

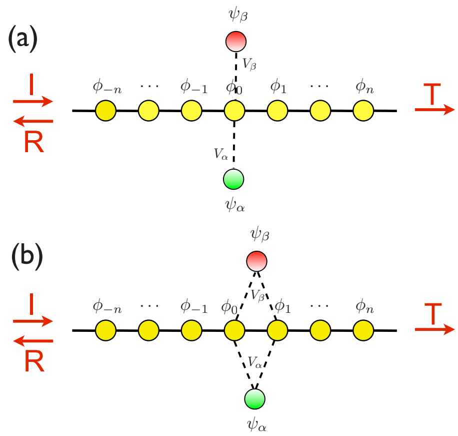

FIG. 1. (Color online) Schematical view of a one-dimensional discrete system with two (a) local and (b) nonlocal side-coupled defects.

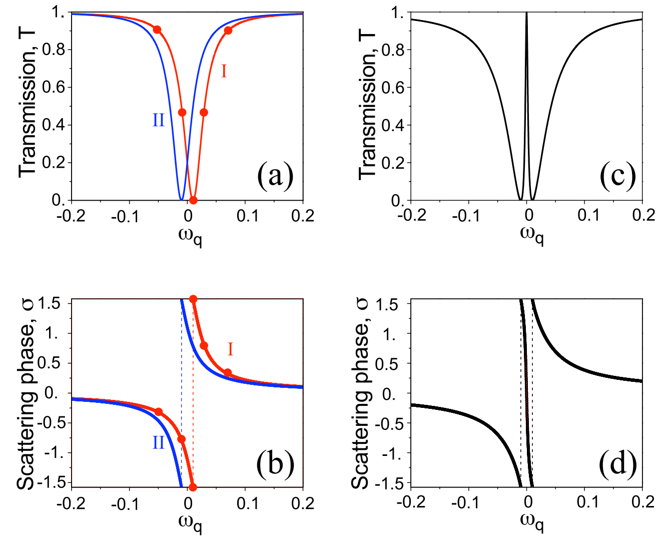

FIG. 2. (Color online) (a) Transmission coefficients T, and (b) scattering phases $\sigma = \arg t$ for single side-coupled defect for two cases: (i) $V_\alpha=0.2$ and $V_\beta=0$ (red curve, I), and (ii) $V_\alpha=0$ and $V_\beta=0.2$ (blue curve, II). (c) Transmission coefficient and (d) scattering phase of two interacting resonances $V_\alpha=V_\beta=0.2$ (black curve). The eigenfrequencies of side-coupled defects are $E_\alpha=-E_\beta=0.01$.

We start our analysis by considering the local coupling regime, where two defects are coupled only to one site of the discrete array

$$\omega_q\phi_n = \phi_{n-1} + \phi_{n+1} + [V_\alpha\psi_\alpha + V_\beta\psi_\beta]\delta_{n0}\phi_0,$$

$$\omega_q\psi_\alpha = E_\alpha\psi_\alpha + V_\alpha\phi_0,$$

$$\omega_q\psi_\beta = E_\beta\psi_\beta + V_\beta\phi_0, \tag{1}$$

where $\omega_q = 2\cos q$ is the dispersion of the system, and use the following boundary conditions:

$$\phi_n = \begin{cases} Ie^{iqn} + re^{-iqn}, & n<0, \\ te^{iqn}, & n>0, \end{cases} \tag{2}$$

for the scattering problem (Fig. 1).

It is known that for single side-coupled defect, i.e., when either $V_\alpha=0$ or $V_\beta=0$, the system (1) exhibits a single Fano resonance [3], where the total transmission $T=|t/I|^2$ vanishes exactly at the eigenfrequency of one of the side-coupled defect $T(\omega_q=E_{\alpha,\beta})=0$ [4] [see Fig. 2(a)]. This resonance originates from the destructive interference of the incoming wave and resonantly excited radiation of the side-coupled defect. Figure 2(b) indicates π-jump of the phase of the scattering wave, $\sigma=\arg t$, at the resonance, leading to the total destructive interference.

In the case of two side-coupled defects ($V_\alpha \neq 0$ and $V_\beta \neq 0$), the transmission coefficient can be easily found analytically by substituting $\psi_\alpha = V_\alpha\phi_0/(\omega_q - E_\alpha)$ and $\psi_\beta = V_\beta\phi_0/(\omega_q - E_\beta)$ into the first equation in (1), and using the general transfer matrix approach [4]

$$T = \frac{4\sin^2 q\Delta\omega_\alpha^2\Delta\omega_\beta^2}{4\sin^2 q\Delta\omega_\alpha^2\Delta\omega_\beta^2 + (V_\alpha^2\Delta\omega_\beta + V_\beta^2\Delta\omega_\alpha)^2}, \tag{3}$$

where $\Delta\omega_\alpha = \omega_q - E_\alpha$ and $\Delta\omega_\beta = \omega_q - E_\beta$.

One can see, that in this case the transmission coefficient (3) vanishes exactly at two frequencies $\Delta\omega_{\alpha,\beta}=0$, which correspond to eigenfrequencies of side-coupled defects $\omega_q^{T=0} = E_{\alpha,\beta}$. In addition to that, there is a perfect transmission between these two total reflections [see Fig. 2(c)]

$$\omega_q^{T=1} = (V_\alpha^2 E_\beta + V_\beta^2 E_\alpha)/(V_\alpha^2 + V_\beta^2). \tag{4}$$

It appears as a result of interaction of two overlapping Fano resonances, where the distance between resonances is smaller then their width, due to two subsequent π-jumps of the scattering phase with an intermediate zero value $\sigma=0$ [see Fig. 2(d)]. It leads to constructive interference condition for incoming and scattered waves, given rise to an addition total transmission resonance. The width of this resonance depends on the distance between two Fano resonances $\Delta\omega = |\omega_q^{T=1} - \omega_q^{T=0}| \approx |E_\alpha - E_\beta|$, and may become extremely small $\Delta\omega \ll 0$ if $E_\alpha \approx E_\beta$. The similar results were first derived by Feshbach [29,30], in application to generic overlapping (or interacting) resonances, which allows us to call them as the Fano-Feshbach resonances in this case. Some examples of this kind of resonance are the electromagnetically induced transparency, in various atomic and optical systems [32–35], phase-coherent electron transport in open quantum systems, consisting of nanowire and two side-coupled quantum dots [36], electronic waveguide with two symmetrically attached stubs [37], etc.

III. NONLOCAL COUPLING

As a next step of our study, we will consider the nonlocal coupling regime, where two side-coupled defects are coupled to two nearest sites of the one-dimensional discrete array [see Fig. 1(b)]. As we have shown in Ref. [4], the nonlocal coupling, in general, leads to renormalization of the position of the Fano resonances $\omega_q^{T=0} \neq E_{\alpha,\beta}$. In this section we will investigate how resonances will be shifted, and how it will affect their interaction. First, we will consider the linear regime, where the transmission coefficient can be still found analytically. Then, we will add a nonlinearity to one of the side-coupled defects, and study the nonlinear transmission.

A. Linear regime

The model with nonlocal couplings to two nearest sites $n=0,1$ can be written as follows:

$$\omega_q\phi_n = \phi_{n-1} + \phi_{n+1} + \sum_{m=0,1} (V_\alpha^m\psi_\alpha + V_\beta^m\psi_\beta)\delta_{nm},$$

$$\omega_q\psi_\alpha = E_\alpha\psi_\alpha + V_\alpha^0\phi_0 + V_\alpha^1\phi_1,$$

$$\omega_q\psi_\beta = E_\beta\psi_\beta + V_\beta^0\phi_0 + V_\beta^1\phi_1. \tag{5}$$

The general expressions for transmission and reflection coefficients are quite cumbersome. So, we will present them here only for a symmetric case, where we assume that couplings to both sites are equal $V_\alpha \equiv V_\alpha^0 = V_\alpha^1$ and $V_\beta \equiv V_\beta^0 = V_\beta^1$,

$$T = \left|\frac{(e^{iq}-1)(a+b)}{(1-e^{-iq})a+2b}\right|^2, \quad R = \left|\frac{(1+e^{iq})b}{(1-e^{-iq})a+2b}\right|^2. \tag{6}$$

where $a=(\omega_q-E_\alpha)(\omega_q-E_\beta)$, and $b=(V_\alpha^2+V_\beta^2)\omega_q - V_\beta^2E_\alpha - V_\alpha^2E_\beta$.

From Eq. (6) one can formally derive conditions for perfect reflection $a+b=0$ ($T=0$), and perfect transmission $b=0$ ($R=0$). Note here, that both conditions become nearly identical when $a \ll 0$, i.e., $\omega_q \approx E_{\alpha,\beta}$. As a result, one may expect the appearance of the extremely narrow asymmetric resonances near side-coupled defects eigenfrequencies, where the total transmission and total reflections will be located very close to each other.

The condition for total transmission $b=0$ gives us the same expression as in the case of local coupling regime Eq. (4), which means that there is no shift of the resonant transmission due to nonlocality. Note here, that there is one more additional condition for perfect transmission ($R=0$) in Eq. (6), which is $1+e^{iq}=0$, or simply $q=\pi$. In this case, the incoming wave has the staggered form $\phi_n=(-1)^n$. As a result, in the symmetric coupling case $V_{\alpha,\beta}^0 = V_{\alpha,\beta}^1$ the equations for side-coupled defects $\psi_{\alpha,\beta}$ Eq. (5) become effectively decoupled from the discrete system, since $\phi_0+\phi_1=0$, resulting in fully transparent system without any defects.

The condition for perfect reflection $a+b=0$ ($T=0$) gives us, in general, two solutions

$$\omega_q^{T=0} = -\frac{V_\alpha^2+V_\beta^2-E_\alpha-E_\beta \pm \sqrt{(V_\alpha^2+V_\beta^2)^2+(E_\alpha-E_\beta)(2V_\beta^2-2V_\alpha^2+E_\alpha-E_\beta)}}{2}. \tag{7}$$

For a particular realistic situation, it will be useful to consider the situation of equal coupling coefficients $V \equiv V_\alpha = V_\beta$, which will drastically simplify the expressions for total transmission (4),

$$\omega_q^{T=1} = \frac{E_\alpha+E_\beta}{2}, \tag{8}$$

and total reflections (7)

$$\omega_q^{T=0} = -\frac{2V^2-E_\alpha-E_\beta \pm \sqrt{[4V^4+(E_\alpha-E_\beta)^2]}}{2}. \tag{9}$$

The typical profile of the transmission coefficient in the nonlocal regime is shown on Fig. 3(a), where there are two resonant reflections and one resonant transmission in between. Due to renormalization, the total transmission is much closer to one of the total reflection, forming the single sharp asymmetric Fano resonance [3,4]. For this type of asymmetric resonance one can calculate its width as follows:

$$\Delta\omega = |\omega_q^{T=1} - \omega_q^{T=0}| = |V^2 - \sqrt{V^4+(\Delta E)^2/4}|, \tag{10}$$

where $\Delta E = E_\alpha - E_\beta$. The width of the resonance become extremely small $\Delta\omega \ll 1$, when two side-coupled defects are nearly identical $E_\alpha \approx E_\beta$. This behavior is similar to the local coupling regime, except that there is second much wider and well separated total reflection resonance (see Fig. 3). Remarkably, in the situation of two identical side-coupled defects $E_\alpha = E_\beta$ the sharp asymmetric resonance disappears since $\omega_q^{T=1} = \omega_q^{T=0}$ [17]. In the literature such a situation is often associated with the appearance of the bound state in the continuum [36,38], first proposed by von Neumann and Wigner [39].

Figure 4 illustrates how the renormalization happens as we increase the degree of nonlocality $\rho = V_{\alpha,\beta}^1/V_{\alpha,\beta}^0$ from zero to one. It can be clearly seen, that the total transmission does not shift at all, while the total reflections do change their positions. Initially symmetric distribution of resonances becomes asymmetric, where one of the total reflections moves away and another approaches very closely the total transmission, resulting in a sharp asymmetric resonance. This figure suggests that for the same parameters the nonlocal coupling regime provides the narrower resonance, i.e., with larger quality factor, compared to the local coupling regime. It makes the nonlocal regime more attractive for possible applications.

B. Nonlinear resonances

It is possible to obtain the analytical expressions for excitation of the side-coupled defects

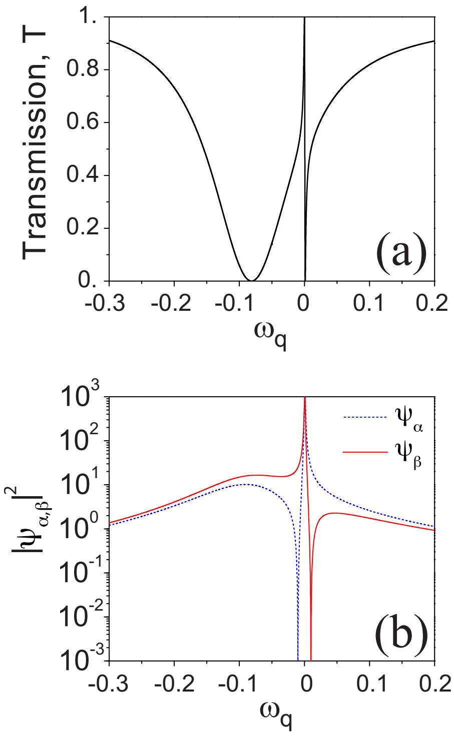

FIG. 3. (Color online) (a) Transmission coefficient (6) and (b) side-coupled defects excitations (11) for nonlocal case for parameters $V_\alpha = V_\beta = 0.2$ and $E_\alpha = -E_\beta = 0.01$.

$$\psi_\alpha = -\frac{2ie^{iq}\sin qV_\alpha\Delta\omega_\beta}{(1-e^{-iq})a+2b},$$

$$\psi_\beta = -\frac{2ie^{iq}\sin qV_\beta\Delta\omega_\alpha}{(1-e^{-iq})a+2b}. \qquad (11)$$

Figure 3(b) clearly indicates that near the sharp asymmetric resonance both side-coupled defects are highly excited. If now we add a nonlinearity, one may expect the appearance of nonlinear effects at very low input powers. For sake of simplicity we first add a Kerr type of nonlinearity to one side-coupled defect only in Eq. (5), say ψ_α,

$$\omega_q\psi_\alpha = E_\alpha\psi_\alpha + \lambda|\psi_\alpha|^2\psi_\alpha + V_\alpha(\phi_0+\phi_1), \qquad (12)$$

where λ is the nonlinear coefficient.

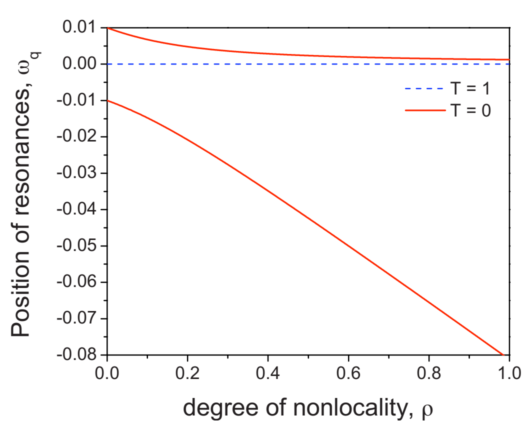

FIG. 4. (Color online) Dependence of the position of the total transmission $T=1$ (blue dashed curve), and total reflection $T=0$ (red solid curve) resonances versus degree of nonlocality $\rho = V^1_{\alpha,\beta}/V^0_{\alpha,\beta}$, showing the renormalization of resonances from local case $\rho=0$ [see Fig. 2(c)] to nonlocal one $\rho=1$ [see Fig. 3(a)].

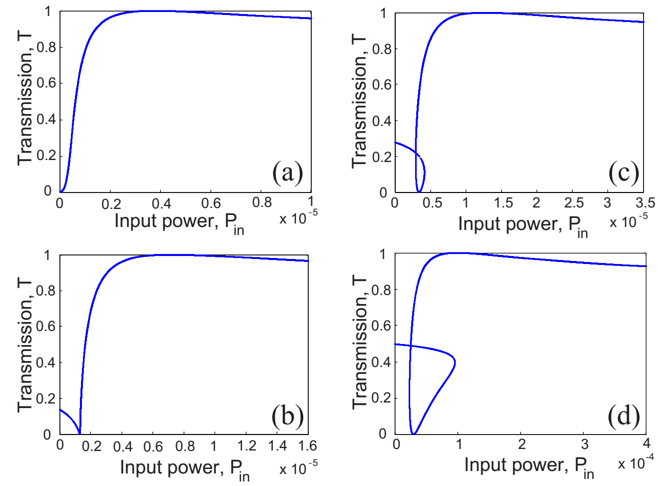

FIG. 5. (Color online) Nonlinear transmission coefficient in the nonlocal case for parameters from Fig. 3, $\lambda=1$, and fixed frequency: (a) $\omega_q=0.0012$, (b) $\omega_q=0.002$, (c) $\omega_q=0.003$, and (d) $\omega_q=0.01$.

Some typical nonlinear transmission profiles are shown in Fig. 5, where both resonant transmission and resonant reflection can be observed on the same curves. The variation of the transmission from zero to one is the intrinsic characteristic of the nonlinear Fano-Feshbach resonance. It can be very useful in some realistic applications, for example, in all-optical switchings in photonic crystals [17]. This figure also illustrates one of the peculiarities of this type of resonance namely, the formation of a closed loop, which results in the bistable transmission. This feature is quite unique for this type of resonance. Since we have chosen the frequencies of the incoming wave in the vicinity of the sharp asymmetric resonance, the threshold power is very low, orders of 10^{-5}.

We have also considered the situation when the nonlinearity was added to both side-coupled defect. In general, the situation does not change much, but the nonlinear transmission coefficient becomes even more complicated with double- and triple-folded closed loops. It makes it a bit harder to read the data, and they are not shown here.

IV. CONCLUSIONS

We have studied interaction of two Fano resonances using a simple one-dimensional model with two side-coupled defects. Each side-coupled defect leads to appearance of a single Fano resonance in the form of resonant suppression of transmission. The presence of two nearly identical defects leads to the interaction of two closely located Fano resonances, resulting in the total transmission resonance, due to the phase cancelation. We have considered two different cases of local and nonlocal coupling, and demonstrated that the latter case provides with a sharp asymmetric resonance of larger quality factor than the former case for the same parameters. By adding a nonlinearity to one of the defects, it is possible to achieve a bistable response at very low input powers with a closed loop in the nonlinear transmission curve.

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council. The author thanks Sergei Mingaleev and Yuri Kivshar for useful discussions.

[1] R. W. Wood, Philos. Mag. **4**, 396 (1902).
[2] A. Hessel and A. A. Oliner, Appl. Opt. **4**, 1275 (1965).
[3] U. Fano, Phys. Rev. **124**, 1866 (1961).
[4] A. E. Miroshnichenko, S. F. Mingaleev, S. Flach, and Yu. S. Kivshar, Phys. Rev. E **71**, 036626 (2005).
[5] K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, Phys. Rev. Lett. **88**, 256806 (2002).
[6] J. U. Nöckel and A. D. Stone, Phys. Rev. B **50**, 17415 (1994).
[7] J. Göres, D. Goldhaber-Gordon, S. Heemeyer, M. A. Kastner, H. Shtrikman, D. Mahalu, and U. Meirav, Phys. Rev. B **62**, 2188 (2000).
[8] B. R. Bulka and P. Stefanski, Phys. Rev. Lett. **86**, 5128 (2001).
[9] M. E. Torio, K. Hallberg, S. Flach, A. E. Miroshnichenko, and M. Titov, Eur. Phys. J. B **37**, 399 (2004); A. A. Aligia and L. A. Salguero, Phys. Rev. B **70**, 075307 (2004).
[10] S. Noda, A. Chutinan, and M. Imada, Nature (London) **407**, 608 (2000).
[11] S. H. Fan and J. D. Joannopoulos, Phys. Rev. B **65**, 235112 (2002).
[12] S. H. Fan, W. Suh, and J. D. Joannopoulos, J. Opt. Soc. Am. A **20**, 569 (2003).
[13] S. Flach, V. Fleurov, A. V. Gorbach, and A. E. Miroshnichenko, Phys. Rev. Lett. **95**, 023901 (2005).
[14] A. E. Miroshnichenko and Yu. S. Kivshar, Opt. Express **13**, 3969 (2005).
[15] S. F. Mingaleev, A. E. Miroshnichenko, Yu. S. Kivshar, and K. Busch, Phys. Rev. E **74**, 046603 (2006).
[16] S. F. Mingaleev, A. E. Miroshnichenko, and Yu. S. Kivshar, Opt. Express **15**, 12380 (2007).
[17] S. F. Mingaleev, A. E. Miroshnichenko, and Yu. S. Kivshar, Opt. Express **16**, 11647 (2008).
[18] E. Tekman and P. F. Bagwell, Phys. Rev. B **48**, 2553 (1993).
[19] S. W. Kim and S. Kim, Phys. Rev. B **63**, 212301 (2001).
[20] S. Flach, A. E. Miroshnichenko, V. Fleurov, and M. V. Fistul, Phys. Rev. Lett. **90**, 084101 (2003).
[21] A. Micheli, A. J. Daley, D. Jaksch, and P. Zoller, Phys. Rev. Lett. **93**, 140408 (2004).
[22] A. Micheli and P. Zoller, Phys. Rev. A **73**, 043613 (2006).
[23] R. A. Vicencio, J. Brand, and S. Flach, Phys. Rev. Lett. **98**, 184102 (2007).
[24] M. F. Yanik, S. H. Fan, and M. Soljačić, Appl. Phys. Lett. **83**, 2739 (2003).
[25] A. R. Cowan and J. F. Young, Phys. Rev. E **68**, 046606 (2003).
[26] V. Lousse and J. P. Vigneron, Phys. Rev. B **69**, 155106 (2004).
[27] A. E. Miroshnichenko, Yu. S. Kivshar, R. A. Vicencio, and M. I. Molina, Opt. Lett. **30**, 872 (2005).
[28] B. Maes, P. Bienstman, and R. Baets, Opt. Express **16**, 3069 (2008).
[29] H. Feshbach, Ann. Phys. (N.Y.) **5**, 357 (1958); **19**, 287 (1962).
[30] F. H. Mies, Phys. Rev. **175**, 164 (1968); J. Chem. Phys. **51**, 787 (1969).
[31] S. Fan, Physica B **394**, 221 (2007).
[32] D. D. Smith, H. Chang, K. A. Fuller, A. T. Rosenberger, and R. W. Boyd, Phys. Rev. A **69**, 063804 (2004).
[33] Q. Xu, S. Sandhu, M. L. Povinelli, J. Shakya, S. Fan, and M. Lipson, Phys. Rev. Lett. **96**, 123901 (2006).
[34] Q. Xu, J. Shakya, and M. Lipson, Opt. Express **14**, 6463 (2006).
[35] R. W. Boyd and D. J. Gauthier, Nature (London) **441**, 701 (2006).
[36] O. Valsson, C.-S. Tang, and V. Gudmundsson, Phys. Rev. B **78**, 165318 (2008).
[37] C. S. Lent, Appl. Phys. Lett. **57**, 1678 (1990).
[38] F. H. Stillinger and D. R. Herrick, Phys. Rev. A **11**, 446 (1975).
[39] J. von Neumann and E. Wigner, Z. Phys. **30**, 465 (1929).